

Man bilde die erste Ableitung y' :

1. $y = x^2 - 3x^4 + 6x^5$;

2. $y = x^{\frac{7}{5}}(x^3 - \frac{2}{x} + 6) + (\sqrt{x^3 - 2})^7 + 26$;

3. $y = \frac{(a^2 + x^2)^3}{(a^3 + x^3)^2}$;

4. $y = \frac{x}{x + \sqrt{a^2 + x^2}}$;

5. $y = \sqrt{a+x} \cdot \sqrt[3]{a^2 - x^2}$;

6. $y = \frac{x^2}{\cos x}$;

7. $y = \frac{x}{4} \cos \frac{\pi}{3}$;

8. $y = x(\arcsin x)^2$;

9. $y = \arcsin \sqrt{\frac{1-x}{1+x}}$;

10. $y = \arccos(2 \tan x)$;

11. $y = a \arcsin \frac{x}{a} - \sqrt{a^2 - x^2}$, $|x| < a$;

12. $y = \tan \sqrt[3]{2 - \sin^2 x}$;

13. $y = \sin^2 x^2 - \cos \sqrt{x} + \cot(\tan x)$;

14. $y = |x^2 - 1| - |x|$;

15. $y = |\sin x| + \cos x$;

16. $y = \frac{1}{\tan |x|}$;

17. $y = \arctan x + \operatorname{arccot} x$;

18. $y = \cos[\sin(\cos x)]$;

19. $y = \arctan \frac{1}{\sqrt{x}}$;

20. $y = \arcsin \sin |x|$;

21. $y = xe^x$;

22. $y = \ln(xe^x)$;

23. $y = \sin \ln x + \cos \ln x$;

24. $y = \ln(e^{2x} + e^{x^2} - xe^{\sin x})$;

25. $y = 3^x - \tan \ln x$;

26. $y = 7^{\sin x} - 1^{\cos x}$;

27. $y = e^{x^{\ln x}} - x^{\ln x}$;

28. $y = x^x - x^{x^2}$;

29. $y = \sin^2 e^x + 3 - \tan^2 e^{-x}$;

30. $y = \frac{x^2}{10^x} - \sin \frac{1}{x}$;

31. $y = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$;

32. $y = \ln |x + \sqrt{x^2 + a}|$;

33. $y = \ln |x - \sqrt{x^2 + a}|$;

34. $y = -\frac{1}{2}e^{\frac{1}{x}} + \sinh x$;

35. $y = \cosh^2 \sqrt{x} - \sinh^2 \sqrt{x} + 3x$;

36. $y = \frac{(\ln |x|)^n}{x}$;

37. $y = \cos \cosh x - \tanh \tan x$;

38. $y = \operatorname{artanh} \sin x$;

39. $y = \operatorname{arsinh} x^2 - \operatorname{arcosh} x^2$;

40. $y = \ln(\ln x^2)^2 - \tan \frac{\ln x}{x}$;

41. $y = \sqrt{\sin x}$;

42. $y = \ln \sqrt{\cos x}$;

43. $y = e^{\sin x}$;

44. $y = x^2 e^{-x} + 2x e^x - x^2$;

45. $y = \cos x$;

46. $y = \cosh x - \sinh x$;

47. $y = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$;

48. $y = |x^3|$;

49. $y = x \ln x$;

50. $y = \frac{1}{|x|} - \frac{1}{x}$;

Für Nr. 45–50 bilde man die n . Ableitung $y^{(n)}$.

Ergebnisse: (mit $\text{sign } x = |x|/x$ für $x \neq 0$)

1) $2x - 12x^3 + 30x^4$

3) $\frac{6a^2x(a-x)(a^2+x^2)^2}{(a^3+x^3)^3}$

5) $\frac{(a+x)(3a-7x)}{6\sqrt{a+x}\sqrt[3]{(a^2-x^2)^2}}$

7) $\frac{1}{8}$

9) $\frac{-1}{(1+x)\sqrt{2x-2x^2}}$

11) $\sqrt{\frac{a+x}{a-x}}$

13) $2x \sin(2x^2) + \frac{\sin \sqrt{x}}{2\sqrt{x}} - \frac{1+\tan^2 x}{\sin^2(\tan x)}$

15) $\sin x \left(\frac{\cos x}{|\sin x|} - 1 \right)$

17) 0

19) $\frac{-1}{2(x+1)\sqrt{x}}$

21) $e^x(1+x)$

23) $\frac{\cos(\ln x)}{x} - \frac{\sin(\ln x)}{x}$

25) $(\ln 3)3^x - \frac{1}{x}(1+\tan^2(\ln x))$

27) $2(e^{x^{\ln x}} - 1)x^{\ln x - 1} \ln x$

29) $e^x \sin(2e^x) + \frac{2e^{-x} \cdot \tan(e^{-x})}{\cos^2(e^{-x})}$

31) $\frac{1}{x^2 - a^2}$

33) $\frac{-1}{\sqrt{x^2 + a}}$

35) 3

37) $-\sinh x \cdot \sin(\cosh x) - \frac{1}{\cosh^2(\tan x) \cdot \cos^2 x}$

39) $2x \left[\frac{1}{\sqrt{1+x^4}} - \frac{1}{\sqrt{x^4-1}} \right]$

41) $\frac{\cos x}{2\sqrt{\sin x}}$

43) $(\cos x)e^{\sin x}$

45) $y^{(n)} = \cos \left(x + n\frac{\pi}{2} \right)$

47) $y' = 1 + x + \frac{x^2}{2}, \quad y'' = 1 + x,$

$y''' = 1, \quad y^{(n)} = 0, \quad n \geq 4$

49) $y' = 1 + \ln x, \quad y^{(n)} = \frac{(-1)^n(n-2)!}{x^{n-1}}, \quad n \geq 2.$

2) $\frac{34}{9}x^{\frac{25}{9}} + \frac{4}{9}x^{-\frac{11}{9}} + \frac{14}{3}x^{-\frac{2}{9}} + \frac{21}{2}\sqrt{x}(\sqrt{x^3}-2)^6$

4) $\frac{a^2}{\sqrt{a^2+x^2}(\sqrt{a^2+x^2}+x)^2}$

6) $\frac{2x}{\cos x} + \frac{x^2 \sin x}{\cos^2 x}$

8) $(\arcsin x)^2 + \frac{2x \arcsin x}{\sqrt{1-x^2}}$

10) $\frac{-2(1+\tan^2 x)}{\sqrt{1-4\tan^2 x}}$

12) $\frac{-\sin 2x}{3 \cos^2 \left(\sqrt[3]{2-\sin^2 x} \right) \sqrt[3]{(2-\sin^2 x)^2}}$

14) $2x \text{sign}(x^2-1) - \text{sign } x$

16) $\frac{-\text{sign } x}{\sin^2 |x|}$

18) $\sin(\sin(\cos x)) \cdot \cos(\cos x) \cdot \sin x$

20) $\text{sign } x$

22) $1 + \frac{1}{x}$

24) $\frac{2e^{2x} + 2xe^{x^2} - (1+x \cos x)e^{\sin x}}{e^{2x} + e^{x^2} - xe^{\sin x}}$

26) $(\ln 7)(\cos x)7^{\sin x}$

28) $(1+\ln x)x^x - (2x \ln x + x)x^{x^2}$

30) $\frac{2x - x^2 \ln 10}{10^x} + \frac{\cos \frac{1}{x}}{x^2}$

32) $\frac{1}{\sqrt{x^2+a}}$

34) $\frac{1}{2x^2}e^{\frac{1}{x}} + \cosh x$

36) $\frac{n(\ln |x|)^{n-1} - (\ln |x|)^n}{x^2}$

38) $\frac{1}{\cos x}$

40) $\frac{4}{x \ln x^2} - \frac{1 - \ln x}{x^2 \cos^2 \left(\frac{\ln x}{x} \right)}$

42) $-\frac{1}{2} \tan x$

44) $(2x - x^2)e^{-x} + 2(1+x)e^x - 2x$

46) $y^{(n)} = (-1)^n y$

48) $y' = 3x^2 \text{sign } x, \quad y'' = 6x \text{sign } x,$

$y''' = 6 \text{sign } x, \quad y^{(n)} = 0, \quad n \geq 4$

50) $y^{(n)} = \begin{cases} 0 & , x > 0 \\ \frac{(-1)^{n+1} \cdot 2 \cdot n!}{x^{n+1}} & , x < 0 \end{cases}$